ME 314 - Engineering Design : Mechanical Components
Lecture 6

Note Title

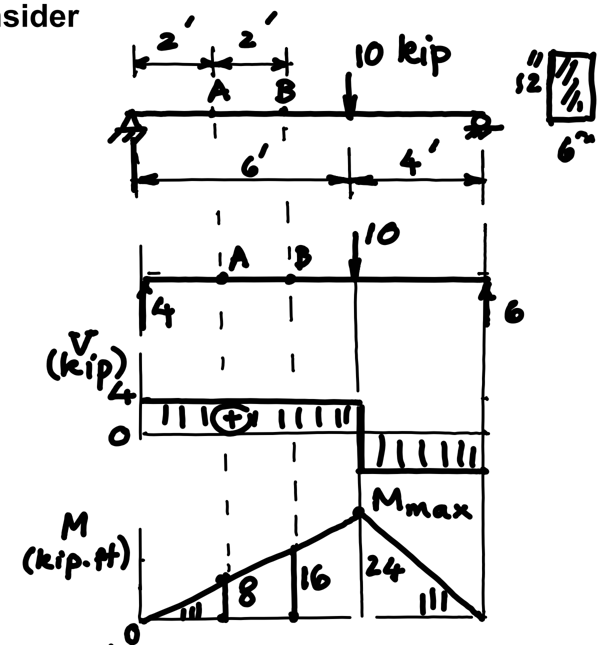
Example: Analyze the shear stress developed in the beam under the loading shown.

Reactions at support and shear & moment diagrams are as shown. To see how shear develops, consider two cross-sections at A & B. We have

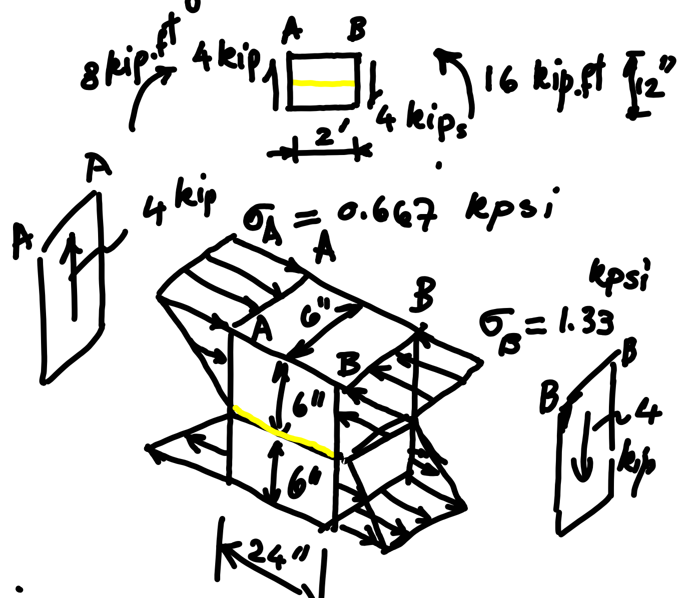
$V_A = V_B = 4$ kips

$M_A = 8$ kip-ft

$M_B = 16$ kip-ft



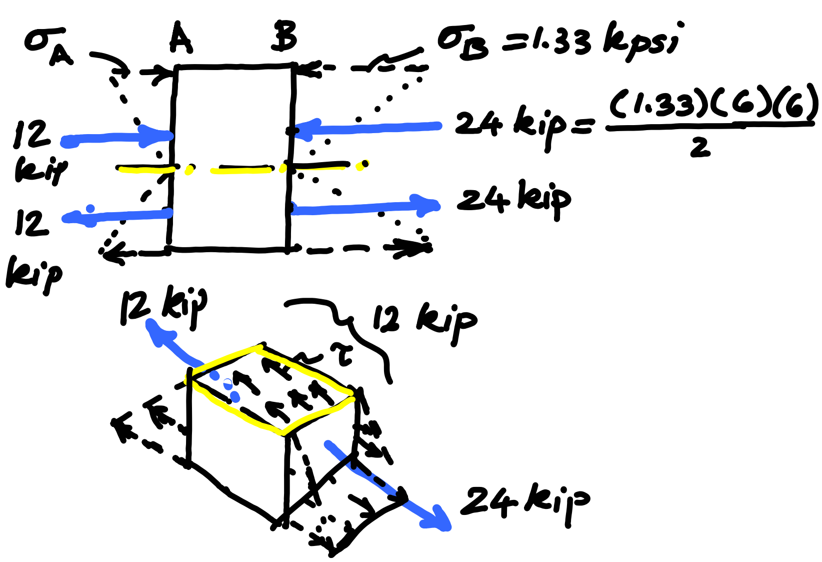
Note that segment AB is in equilibrium, and that

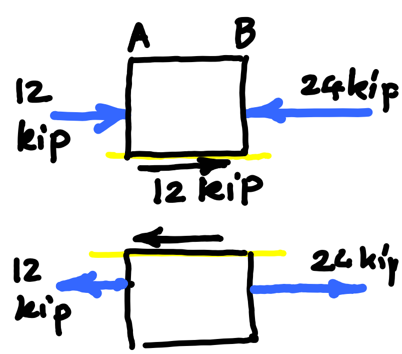


$$\sigma_A = \frac{Mc}{I} = \frac{(8)(12)(6)}{\frac{(6)(12)^3}{12}} = 0.667 \text{ kpsi}$$

$$\sigma_B = 2(0.667) = 1.33 \text{ kpsi}$$

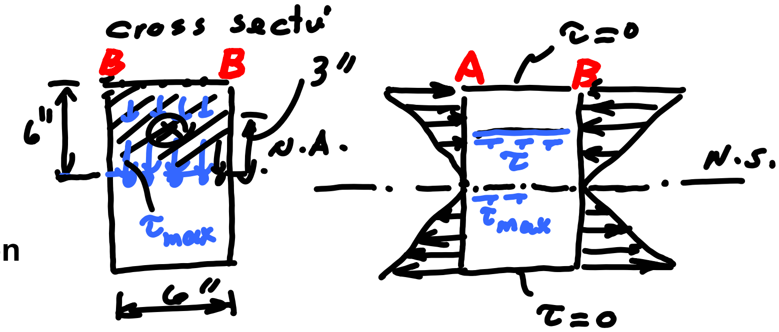
Resultant forces on the cross-sections are shown below:





Position of the cut determines the amount of shear stress. Top & bottom surfaces are under zero shear stresses while the neutral surface is under the maximum shear stress. In Mechanics of Materials, it was shown that

$$\tau = \frac{VQ}{It}$$



Where

Q **= First moment of area above the cut in the cross-section**

$$I = \frac{bh^3}{12} = \frac{(6)(12)^3}{12} = 864 \text{ in}^4$$

At section A or B :

$$\tau_{max} = \frac{(4 \text{ kip})(108 \text{ in}^3)}{(864 \text{ in}^4)(6 \text{ in})} = 83 \text{ psi}$$

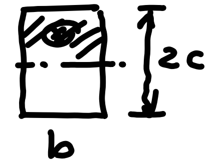
Check shear Force

$$F = (83 \text{ psi})(24 \text{ in})(6 \text{ in}) \simeq 12000 \text{ psi} = 12 \text{ kip}$$

Maximum shear stress for various sections

Rectangular sections:

$$Q = (bc)\left(\frac{c}{2}\right) = \frac{Ac}{4}, \quad I = \frac{b(2c)^3}{12} = \frac{Ac^2}{3}$$



$$\tau_{max} = \frac{VQ}{Ib} = \frac{V(Ac/4)}{(Ac^2/3)(b)} = \frac{3V}{2A}$$

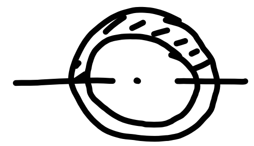
Circular sections:

$$Q = \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right) = \frac{2r^3}{3} \qquad \frac{4r}{3\pi}$$

$$I = \frac{\pi r^4}{4} \qquad \tau_{max} = \frac{VQ}{It} = \frac{4V}{3A}$$

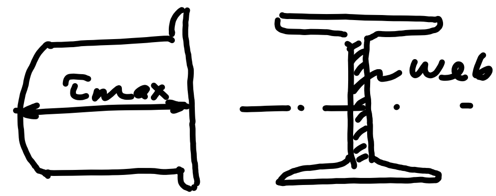
Hollow round:

$$\tau_{max} = \frac{2V}{A}$$



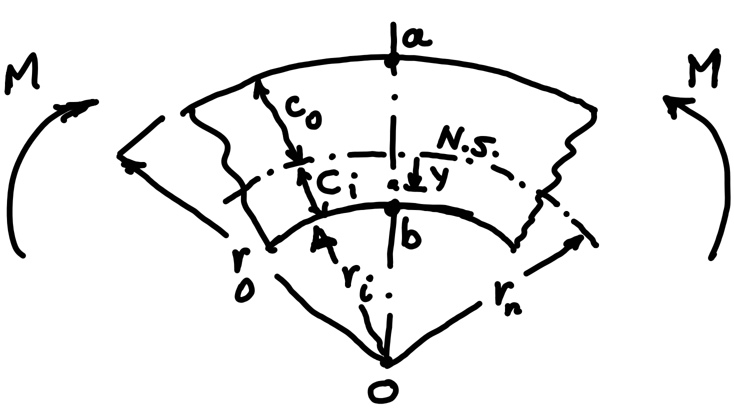
I-Beam:

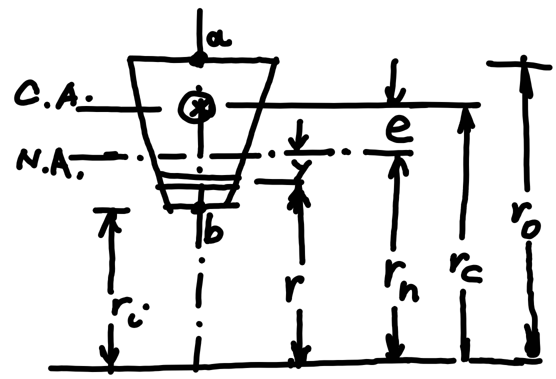
$$\tau_{max} = \frac{V}{A_{web}}$$



Curved Beams

C-clamps & crane hooks are examples of curved beams. To analyze curved beams, we make all the assumptions we made for straight beams except for the last assumption (i.e., # 7).





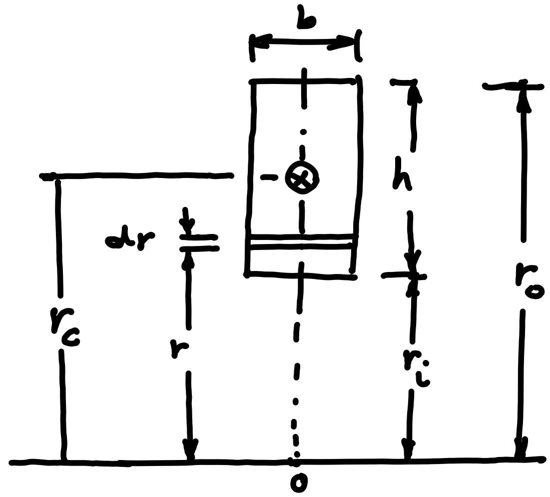
For curved beams the neutral axis (N.A.) is not coincident with the centroidal axis (C.A.). It shifts towards the center of curvature by an amount designated by *e*.

We can show that

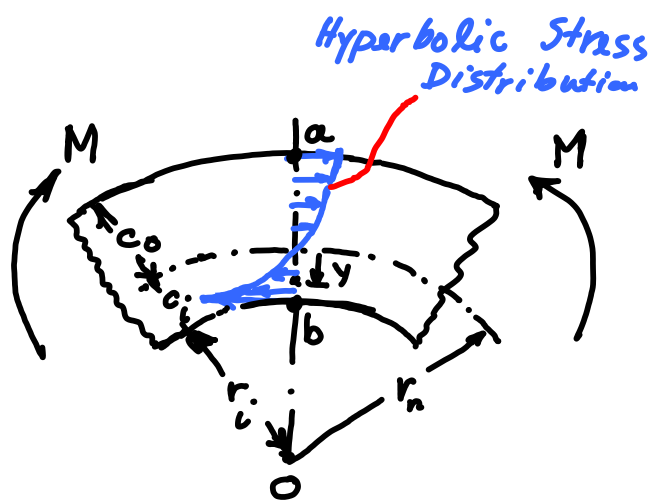
For simple cross-sections r_n could be obtained by integration. Expressions for many common sections are given in the book:

R.J. Roark & W.C. Young, *Formulas for Stress and Strain*. 6th ed. McGraw-Hill, NY 1989.

As an example, consider a rectangular cross-section:



Requiring equilibrium and employing the concepts of strain, and stress-strain relations, we can show that

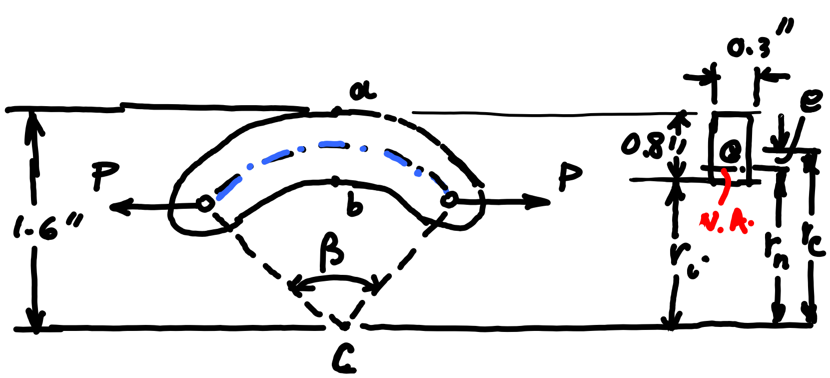


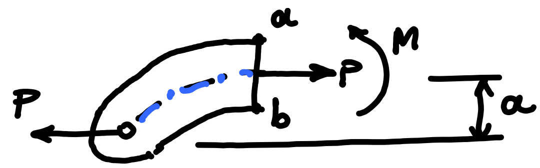
Hence, the distribution of stress is not linear.

Example: Steel links having rectangular cross-section are available with different central angle β. Here take β = 90°. Knowing that the allowable tensile stress is 12 ksi, find the largest load P that may be applied.

Location of neutral axis

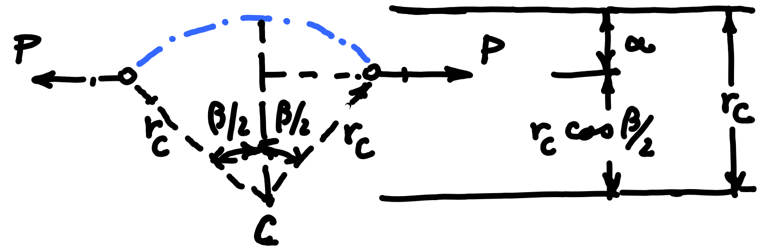
$$r_n = \frac{h}{\ln \frac{r_o}{r_i}}$$





Force-couple at center of section ab:

$$a = r_c - r_c \cos\frac{\beta}{2}$$



$$M = aP$$

Maximum Stress occurs at b:

$$\sigma_{max} = \sigma_b = \frac{P}{A} + \frac{Mc_i}{Aer_i}$$

$$c_i = r_n - r_i$$

$$A =$$

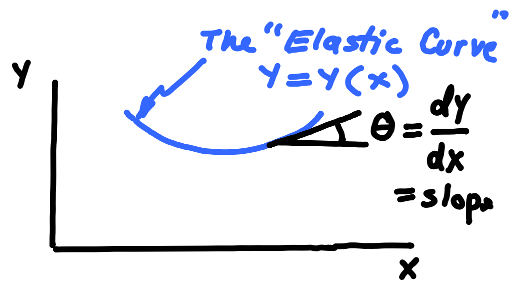
$$r_i = r_o - h =$$

4.10 Deflection in Beams

A beam often requires a larger cross-section to limit deflection than it does to limit stress. Hence, many steel beams are made of low-cost alloys because these have the same modulus of elasticity (thus, the same resistance to elastic deflection) as stronger, high-cost steels. If the beam is sized to prevent stresses that exceed material's yield point then no permanent set (if the material is ductile) or fracture (if the material is brittle) should occur. However, elastic deflections at stresses well below yield stress may still cause serious problems in a machine. For example, they can cause interferences between moving parts or misalignments that destroy the required, accuracy of the device.

The bending deflection of a beam is calculated by integrating the beam equation

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \qquad (4.17)$$



twice to obtain

$$\theta = \frac{dy}{dx} \quad \text{and} \quad Y = Y(x)$$

Equation (4.17) is valid only when the deflections are small.

If the beam is used as a spring then deflections may be large and (4.17) may not apply. Spring design is covered in Chapter 14 of text.

The beam deflection due to transverse shear load is also assumed to be negligible in deriving (4. 17). This makes (4.17) valid only for long beams for which length/depth = L/d > about 10.

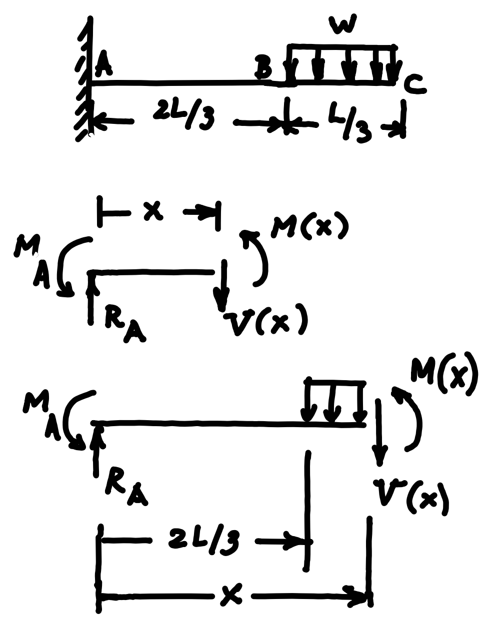
Note that

$$\frac{V}{EI} = \frac{d^3y}{dx^3} \quad \text{and} \quad \frac{q}{EI} = \frac{d^4y}{dx^4}$$

The loading function q = q(x) is typically known and can be integrated four times to obtain y = y(x). Other methods are also used, e.g., superposition method, energy methods (Castigliano's theorem), numerical methods and FEM.

Example: Finding deflections using singularity functions

For the beam loaded and supported as shown in the figure, determine the deflection of the right end.



The constants of integration C_1, C_2, C_3, and C_4 are found from the bounday conditions:

$$V(0)=R_A \quad , \quad M(0)=-M_A \quad , \quad \theta(0)=0 \quad , \quad y(0)=0$$

we have

$$V(0)=R_A=-M_A(0)+R_A(1)-w(0)+C_1 \Rightarrow \underline{C_1=0}$$

$$M(0)=-M_A=-M_A(1)+R_A(0-0)-\frac{w}{2}(0)+0+C_2 \Rightarrow \underline{C_2=0}$$

Remark: As mentioned earlier, when the reactions at supports are included in q(x) as above the constants C_1 and C_2 will be zero.

$$\theta(0) = 0 = \frac{1}{EI}\left[-M_A(0-0) + \frac{R_A}{2}(0-0)^2 - \frac{w}{2}(0) + 0 + 0 + C_3\right] \Rightarrow \underline{C_3 = 0}$$

$$y(0) = 0 = \frac{1}{EI}\left[-\frac{M_A}{2}(0-0)^2 + \frac{R_A}{6}(0-0)^3 - \frac{w}{24}(0) + C_4\right] \Rightarrow \underline{C_4 = 0}$$

Reactions are obtained from Eqs. (2) and (3) by noting that at the free end of the beam ($x = L$) both shear force and moment are zero:

$$V(L) = 0 = -M_A(0) + R_A(1) - w\left(L - \frac{2L}{3}\right) \Rightarrow R_A = \frac{wL}{3} \uparrow$$

$$M(L) = 0 = -M_A(1) + R_A(L-0) - \frac{w}{2}\left(L - \frac{2L}{3}\right)^2$$

$$0 = -M_A + \frac{wL^2}{3} - \frac{wL^2}{18} \Rightarrow M_A = \frac{5wL^2}{18}$$

$$\therefore \quad y = \frac{1}{EI}\left[-\frac{5wL^2}{36}\langle x-0\rangle^2 + \frac{wL}{18}\langle x-0\rangle^3 - \frac{w}{24}\langle x - \frac{2L}{3}\rangle^4\right]$$

and the deflection of the right end is found by setting $x = L$:

$$y(L) = \frac{1}{EI}\left[-\frac{5wL^2}{36}(L)^2 + \frac{wL}{18}(L)^3 - \frac{w}{24}\left(L - \frac{2L}{3}\right)^4\right]$$

$$= \frac{wL^4}{EI}\left[-\frac{5}{36} + \frac{1}{18} - \frac{1}{24(243)}\right] = -\frac{163wL^4}{1944EI} = \frac{163wL^4}{1944EI} \downarrow$$

Singularity functions can also be employed to analyze Statically Indeterminate Beams (see Example 4-7, page 171).